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Reflections on the Walls

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Abstract: Walls are built for a number of reasons among them separation and support. We discuss how our teaching reflects a reflection on just what we are building for our students.

Keywords: Walls, teaching, learning, mathematics.

Introduction

We reflect on our passion for teaching mathematics, useful mathematics, by considering our understandings of just what a wall is and what a wall means to us. Walls separate and keep out. They keep in, but allow for breath through air circulation. Walls seal out things in insulation, yet keep in things like smells. They act as supports. Walls are strong, lest they crumble, but they are also flexible. Walls are to lean upon personally and to organize our space. Simply put, walls are all around us and they shape our world. So why not think of how the notion of a wall can shape our teaching?

In a speech near the Brandenburg Gate on 12 June 1987 President Ronald Reagan, said to the leader of the Soviet Union, Mikhail Gorbachev, "Tear down this wall!"[12]. We build walls for many purposes. In the case of the Berlin Wall it was to contain and separate the East German population from the West German population and keep ideas from flowing. We do not like walls to keep people from moving about, exchanging ideas, or engaging in commerce.

As we pass a new construction site we see the stark concrete of the basement walls in the foundation for the building which is to come. We need walls to support structures and to serve as a foundation. Thus we are not against walls, although we often lean against them!

Robert Frost, in his powerful poem, *Mending Wall*[4, p. 47] opens with this line, "Something there is that doesn't love a wall." In Frost's Yankee contrariness and practicality he is saying we may not love walls, but we know we need them in our lives.

At the beginning of the calculus reform effort [3] in the late 1980's we used the phrase "leaner and livelier"[3] and yet calculus texts are anything but leaner. The lively engagement envisioned by reformers has subsided as we have slipped back into the oblivion of grinding exercises, unengaging prose, an occasional tack-on project activity, and a treatment of our students as juveniles wherein we say, "Just learn this stuff so you can see how it is useful somewhere else, somewhere down the road, in another subject area." Thus our structural wall has to be solid in order to provide a foundation, but it also needs to be leaner, livelier, and flexible.

I have used notions of walls when describing my personal dislike of traditional, encyclopedic texts by telling my students the real value in a tome textbook is that you can slam it against your dorm room wall when yelling at the folks next door to turn down their stereo! I have told my civil engineering students that we could use these building block books to build a real (not virtual) solid wall, a foundation for a learning environments in which we can truly explore mathematics, unencumbered by such texts.

Historically, calculus was used as a filter, not a pump, to keep people from getting into engineering and science studies and by business schools to hold off the hordes by setting up calculus as the roadblock for admission to the programs. To be fair, all of these disciplines used the language of mathematics, but by turning over their students to us in the mathematics departments they were saying, "Help us thin the pack." Thus four years after the "lean and lively report"[3] The Mathematical Association of America published examples of curricula which could help faculty make calculus a pump, not a filter.[10]

We have gone through a wave of some remarkable curriculum reform efforts, but have again returned to traditional texts with but trappings of excitement and very few attempts to reach out and touch cognate disciplines or student's motivational interests. When doing so we have found it to be through an occasional inserted project or end of chapter essay or complex problem set, not through rich motivational opportunities to introduce and support the mathematics under study.

Our structural wall of support needs to permit, indeed encourage, movement of ideas and energies across its barrier. It should not act as a filter, for it needs to give a welcome to new ideas and approaches by not being so dense with dogma, tradition, and contents as to preclude an open investigation, time for reflection, and a reduction in the amount of material thrown at students. Indeed, even though Advanced Placement Calculus course exams have become more open, almost all teachers in AP classes say they have little time for exploration as they have to keep strict timing and march through so much material, much of it algorithmic rather than application based.

Maybe it is time we use different building materials - leaner and livelier compounds which serve to permit diffusion of ideas, indeed, pump them and not filter them. We need to concentrate on a fabric of support for success using lightweight material that has strength, but also flexibility and responsivity. Additionally, maybe we need walls that breathe, that permit the passage of fresh air from applications into breathing tubes and hence brains of our students.

Everything in our society is becoming lighter and stronger; our cars for efficiency, our luggage for ease in use, our technology for an "on the move" generation, and our communication - witness the 140 character limit in Tweeter! Should we not bring our mathematics instruction into this future by making it leaner and livelier, lighter and stronger, applicable and interesting, even fascinating, but certainly engaging. We encourage a re-examination of why we do things and of what purpose our walls serve. Who builds our walls and who calls the shots? How can we partner with our downstream client disciplines who ask us to teach their future students mathematics so as to offer a firm foundation while permitting flexibility and circulation of fresh ideas and approaches?

Leaner and Livelier

If we really are going to make our mathematics instruction leaner and livelier, lighter and more flexible, and more like a pump and less like a filter, then we have to streamline our subject matter and some things have to go and other things have to change.

Let us look at some examples of notions on which we can concentrate less, if at all. We understand that it is often very hard to let go of some of these skills which actually excited us and gave value to our professional role as a mathematician. They certainly drew us in to mathematics, for they served as challenging problems when we were in the youth of our careers.

- *Extraction of square roots by hand* was a valuable skill. We are not being silly here, for we need to recall that this was taught into the 1950's in high school classrooms. We were most fortunate not to have faculty who thought this algorithmic approach was important enough to keep around so students could more fully understood and appreciate the meaning of square root or so they could do square roots in the sand if stranded on a desert island. No, we abandoned this algorithm when we embraced more widely slide rules and then electronic calculators.

- *Completion of the square* as a viable method to solve quadratic equations instead of the quadratic formula which uses this technique once in teacher "watch me" mode and then proffers the concluding quadratic formula for all time use.

- *Logarithm tables and interpolation* were tools for computation, but calculators offered a quick and sure way to do computations so we could concentrate more on process, results, units, applications, and thinking.

- *Trigonometric identities* were a basis for human symbol manipulation as we tried to pair up forms in high school "trig" class derivations. They can be useful at times. For example, the conversion of $\sin(x+y)$ to terms involving only arguments x and y, not $x+y$, can be used in the formal definition for derivative of $f(x)=\sin(x)$ and in changing a sum of sine and cosine term as in $a\sin(\omega t)+b\cos(\omega t)$ in a solution to a second order differential equations into one phase shifted sinusoidal motion which affords a more reasonable understanding of the motion. Then there are half angle formulae for substitution methods on "tricky" integrals. Might we not turn to our computer algebra system (CAS) and ask for assistance, either analytically or numerically, to make these transitions or override them altogether, in the case of the latter half angle formulae in integration?

- *Partial fractions decomposition* in the study of integrals and Laplace transforms did offer an example of a good problem solving strategy, namely, break the problem into smaller more doable pieces and then proceed. However, Mathematica's `Apart` command, for example, can expedite this decomposition and permit students to focus on the essence of the issue, namely that the solution consists of a sum of parts, each with significance, e.g., steady-state and transient solution.

- *Logging to linearize our data* into a straight line mode was appropriate in a paper and pencil world, but now we could do a direct nonlinear least squares fit quite easily in a CAS or a spreadsheet optimization routine.

We need to trim our curriculum of these (and many more) arcane computations, procedures, and derivations. We need to let emerging technologies take on the tedium and detail so we can focus our students' attentions on the broader processes and applications of mathematics.

Thus we still need a good foundation, but we can build it better with students using modern tools and visualization, not just analytic derivations and algorithmic knowledge. Moreover, the learning process itself needs to be leaner, through casting out tedious algorithms and derivations, and livelier through motivating the mathematics under study and tying it to reality as found in cognate disciplines and even bringing in NOW some down stream mathematics.

Building Support Walls. Who is Calling the Shots?

Often we are still teaching algorithms only and covering methods in our mathematics classes because some downstream client is calling the shots. At least we have engaged them to help us design a curriculum for their students. We made the mistake of soliciting and then concurring with the shopping list of topics given to us by these clients, be they school of business, engineering college, or, science faculty individuals or committees. This list is truly a wish list, always way too large for serious consideration, and sometimes twisted beyond our comfort level. We always trim it down, sometimes at our peril for such clients will say, "If you are not going to teach topic A then why did you ask us in the first place?" That is only when they see the syllabus, even before our students get to them. We are damned if we do and damned if we don't. For when we leave out a topic client faculty accost us in the hall or at committee meetings saying, "Did you not teach them such and such?" Even when we do teach the topic, not all students remember every detail and when, in the midst of a long derivation in class, the client discipline professor calls on a particular student to whip out a formula from the past and the student balks, such faculty will gripe about how weak these students are. I know the feeling, for all too often when in an engineering mathematics course I ask students to do a free body diagram, a most basic of skills used in their elementary physics and introductory engineering coursework, to build mathematical equations of phenomena they simply cannot do it. Do I run to the physics department and badger them? No, rather I work with the students, trying to awaken in them the particulars of the skill in our context and build confidence as they see the larger picture in which this skill is central. I try to make them aware of the process in which they can recover, rebuild confidence, find the right tool - perhaps even construct it themselves. I have to use the moment to increase their self-efficacy, their self-confidence, not berate my colleague in physics for a "failure" to teach them such skills properly, if at all.

We should all concur on broader outcomes for our students and one of these is "wherewithal" to solve a problem, be it recall, looking up, asking, using notes, etc. Then it is up to us mathematics folks to give students situations in which they can practice this resourcefulness to build success, and not badger them because they forgot some formula or algorithm. Both mathematics and the client discipline would retain more students if we agreed upon this approach and students would not be caught in the crossfire of invective arrows of accusation as to respective failures to accomplish total competence of certain specific skills.

Consider a curriculum in which you teach mathematics using a computer algebra system (CAS), say Maple or Mathematica. I specifically stick to CAS's which have solid symbol manipulation capabilities, not just numerical power, so students can examine form and structure to see pattern and reinforce theory. You use the CAS for discovery, for seeing the structure of your solutions, for manipulating complex algebra whose by-hand efforts almost always result in student error, the latter causing delay, frustration, and confusion. So you have adopted your CAS to permit success for your students. They have learned sufficient syntax to function. Some students go way beyond this because it is intellectual fun to play with the CAS, to be creative, and to explore the mathematics in light of such a powerful tool - something which might not happen otherwise. In the shopping list your engineering colleagues send back is a request that reads like this. "Students should learn and use the software packages that are standard in engineering practice." Often, for engineers this is MatLab. So you now have a conflict, for you are trying to build on student's familiarity with your CAS which you and your colleagues in mathematics use in order that your students see and learn the mathematics, not the syntax of another software package. Would you consider teaching C++ programming or Pearl in your mathematics courses because the engineers say stick to "...software packages that are standard in engineering practice?" We think not and we would suggest that if these engineering faculty want to teach their students a tool so they can benefit from the mathematics we teach them with a powerful CAS tool in learning then they should do so. However, do not tax our faculty and our students with their particular software constraints for doing engineering at the expense of the energy we have invested in our CAS tool for teaching, learning, and doing mathematics. We have enough to do on their already impossible shopping list.

Other Images of a Wall

Pink Floyd - Are Our Students Just Another Brick in the Wall?

We could selectively pick the lyrics of Pink Floyd's 1979 hit song, "The Wall"[5] particularly lines 2, 3, 4, and 6 of this excerpt. We would prefer to ignore line 1!

> . . . We don't need no education.
>
> We don't need no thought control.
>
> No dark sarcasm in the classroom.
>
> Teachers, leave those kids alone.
>
> Hey, Teacher, leave those kids alone!
>
> All in all you're just another brick in the wall.

Many in the inquiry-based, discovery-based, or inductive approach to teaching mathematics believe that the teacher should ". . . leave those kids alone" as well as be more supportive, thus cutting out "dark sarcasm in the classroom." Every student wants to feel special, feel the personal contact with the teacher and the subject matter, so being ". . . just another brick in the wall" is not a desirable goal for modern teachers. Indeed, some teachers single out students and give names to their inductive conjectures such as "Alice's Lemma" or "Jim's Theorem." This points out the human nature of mathematics and the process of building a constructive wall of support for later mathematics learning and application to other disciplines, a wall built on student discovery and energy.

Consider "off-the-wall" as an expression of unpredictability, perhaps from the notion of the action of a ball in certain sports, say, when the ball bounces "off the wall." Here we think of the wall as a place in which some stability resides and when we come off that wall some "wild and crazy" things, certainly unpredictable ones, can happen. Do we permit our students to be off our wall, say in their own space?

Introducing Situations for "Off-the-wall" Thinking

As an example, consider this situation. Years ago I was in an interdisciplinary team of faculty teaching a first-year curriculum in which a team of 6-8 faculty taught a cohort all their science, engineering, and mathematics coursework[14]. I and the chemist in our group were teaching notions of gases in liquids and my chemist colleague came up with the idea of a lab in which we simply said to them, "Define fizz and come up with an experiment to measure fizz." That was it. Well, we thought these students would get out their Bunsen burners, their flasks, their tubing, their CO_2 generating chemicals, and make carbonated water or something to that effect. Three, shall we say "adventurous" students, disappeared from the lab and after a while I was sent to fetch them. I found them in the computer lab where they had created miniature sound studios with Styrofoam computer cartons and they were recording the sound of fizz from just-popped soda cans. Their definition of fizz was the sound - its intensity and duration - from a newly-opened can of soda. Indeed, they plotted the volume of the sound and used half-life of the decaying sound as their ultimate measure of fizz! Now, we should ask ourselves if we are permitting this notion of the wall, i.e., "off-the-wall," to enter our coursework enough or at all? I would suggest we need to explore ways to do so.

In that same curriculum effort one day I came to class when we were exploring functions of more than one variable and I asked students [15] the following set of questions:

For the function

$$f(x,y) = \frac{(x^3 - 3x + 4)}{(x^4 + 5y^4 + 20)},$$

suppose your eye is precisely on the surface $z = f(x,y)$ (see Figure 1) at the point $(2.8, 0.5, f(2.8, 0.5))$. You look to the left, i.e., in the direction (roughly) $(-1, 0, 0)$. You see a mountain before you.

(a) Determine the point on the mountain which you can see which is nearest to you.

(b) Describe as best you can the points on the mountain which you can see from the point $(2.8, 0.5, f(2.8, 0.5))$.

(c) Determine the amount of surface area on the mountain which you can see from the point $(2.8, .5, f[2.8, .5])$.

From this one activity students had to invent the concepts of partial derivative, tangent plane, gradient vector, orthogonality, and surface area integral, to say nothing of some programming as the surface was not as friendly as it was to be later when I used a similar activity in many other course activities over the years. They had to think "off-the'wall" and certainly out of their box of algorithms and known mathematics.

Are we thinking enough (if at all) of these two notions about walls in our teaching?

Facebook - Telling the World

We add one contemporary notion of a wall.

> On Facebook, social networking site, a wall is a section in your profile where others can write messages to you or leave you gifts, which are icon-like small images. The wall is a public writing space so others who view your profile can see what has been written on your wall. Once you have received a wall message, you can respond directly back to the friend who left it using the "wall-to-wall" mode.[11]

If we are really permitting our students to be creative are we then providing opportunities for them to share that creativity, both the finished product and the process? We need to permit students to communicate through writing assignments in which they detail their technical analysis and reflect on the process. We need to encourage them to share, to post on their wall - whatever that may be for each of them - their new found knowledge and excitement.

There are many ways for such “wall postings." We could encourage peer reading of student papers in our class - our colleagues in English do it to improve writing as do our science colleagues to screen improper approaches in lab reports on a first read. We could have them write a letter to a high school friend (brother or sister) and try to explain what they just learned. We could have class oral presentations or explanations of solutions at the boards for peers. We could push our students to construct electronic places for such sharing, Facebook, Tweeter, Blogs, Wiki’s, etc. We could encourage them to prepare poster sessions at our local, regional, or national student sessions. When is the last time you let your students share their excitement about what they learned with others in ANY manner?

What is it we really want to build and how might we build it?

Do we want to build strict and stiff walls of foundation when we teach our mathematics students? I think not. I believe we want to teach our students how to be competent problem-solvers using the language of mathematics as well as how to be confident problem solution communicators. We really want to build student self-efficacy, i.e. “the belief in one’s capabilities to organize and execute the courses of action required to manage prospective situations.”[1] We do this by putting them into problem situations in which the mathematics can play a productive role in forming and solving the problem and asking them to do just that – solve the problem. There may be some just-in-time reaching out for new mathematics (at least new to them). This affords the best motivation for learning mathematics, for the students say, “I need it and I need it now.” All too often we teach mathematics saying something like, “You will need this when you get to your major course area.” Instead, consider the following idea. In a paper for teachers of American history the distinguished professor of history, Charles G. Sellers (UC Berkeley) forcefully supports this approach:

> The notion that students must first be given facts and then at some distant time in the future will think about them is both a cover-up and a perversion of pedagogy . . . One does not collect facts he does not need, hang on to them, and then stumble across the propitious moments to use them. One is first perplexed by a problem and then makes use of the facts to achieve a solution.[2]

So what does the literature say about this approach of teaching in context, what is often called inductive learning? “Inductive teaching and learning serve as an umbrella term that encompasses a range of instructional methods, including inquiry learning, problem-based learning, project-based learning, case-based learning, discovery learning, and just-in-time teaching."[8, p. 123] Using modeling to teach mathematics is a consummate inductive approach and in a seminal study on inductive teaching and learning in the premier engineering education journal, the American Society for Engineering EducationŠs (ASEE) *Journal of Engineering Education*, the authors conclude that "...inductive methods are consistently found to be at least equal to, and in general more effective than, traditional deductive methods for achieving a broad range of learning outcomes."[8, p. 123] One of the authors, in a closing essay for *PRISM*, the magazine of ASEE, says,

> Another well-entrenched tenet of traditional instruction is the notion that students must first master the underlying principles and theories of a discipline before being asked to solve substantive problems in that discipline. An analysis of the literature (rendered in [8, p. 123]) suggests that there are sometimes good reasons to "teach backwards" by introducing students to complex and realistic problems before exposing them to the relevant theory and equations.[7, p. 55]

Colleagues around the academic world believe in inductive methods that are in touch with the real world and that teach subject matter in context and they are using this approach. I strongly believe we in mathematics education should practice inductive teaching methods for they break down separating walls and thus strengthen the fabric of the supporting walls needed for success for our students.

Learning by Doing in an Active Environment

We all learn by doing in context. It is well-known that while seeing is believing, it is not learning. Listening in and of itself (even attentively) does not produce learning. We need to be active learners, engaged in our own educational experiences and responsible for our own learning. Furthermore, the material being taught is best learned in context. This means that we do not learn well in a vacuum. We learn best when we see a reason, a connection, a context for the immediate material with that which we already know and value. Application of subject matter motivates learning, whether it is writing a school newspaper piece to learn paragraph structure, learning optimization mathematics in order to maximize yield in a field activity, studying history to learn from our mistakes, or experiencing a laboratory phenomenon to support a physical law or principle.

Building confidence is the primary rationale in permitting students to engage in their own learning. All too often, in their past, students who currently struggle with mathematics have been beaten down by mathematics and been unsuccessful at learning algorithmic material in an unmotivated setting. I have seen students build confidence, high-fiving in a mathematics class, and going on to build for success in their understanding and use of mathematics. I have had students say, "What do you want me to do?" in the beginning of my course say, "I get it and know what to ask of myself now." That's confidence! My goal in teaching is to get my students to address the main issue of problem-solving as stated by George Polya, to know "what to do when you do not know what to do"[6] and to be confident in their approach once they have addressed the problem.

It just might be time for us to bring down the walls of Jericho as Joshua did with his Israelite followers in the Bible. The walls we might want to bring down defend old ideas, worship arcane practices, and limit imagination, especially in light of possible uses of technology and modeling opportunities to motivate the study of mathematics.

An essential added ingredient I seek to generate is student interest, for it is essential to have this present in students. Paul J. Silvia, a social psychologist, has determined that, "When interested, students persist longer at learning tasks, spend more time studying, read more deeply, remember more of what they read, and get better grades in their classes." Furthermore, "In the case of interest, people are 'dealing with' an unexpected and complex event – they are trying to understand it. In short, if people appraise an event as new and as comprehensible, then they will find it interesting." Finally, "finding something understandable is the hinge between interest and confusion . . . ."[9, p. 58] Fostering student interest is essential for learning.

Bottom Line Up Front - we first nurture interest in our students' minds and as they build confidence they learn best through active participation and application of the material under study.

Caveats and Personal Losses

Can we only teach broad skills and approaches? No, we need to get down to the nitty gritty of our discipline, we need to do some derivations, but we need, most of all, to motivate the mathematics we teach in a context which is supportive and focused on why we are doing what we are doing with immediacy (not down the road) as the operative word. We are not producing a bunch of feel-gooders, of problem managers, of white board diagrammers. We are equipping our students with mathematical knowledge, tools, understanding and comprehension, and experiences so they can contribute to their own learning in downstream disciplines which value mathematics and what it has to offer their own recently adopted discipline, be it economics, engineering, science, etc.

We understand that some of the techniques and algorithms we might be setting aside or downgrading in terms of time spent on task could be some of our personal favorites, our personal intellectual high point for discovery or motivation. These could be the very stuff of what drew us into mathematics when we were younger. As an example, consider how you felt when programs dropped many methods of integration and you had to fight with the folks who argued for use of integral tables or computer algebra systems. The nuances and subtleties of classifying and then applying techniques to integrals as they came your way was enjoyable, you felt good about your accomplishments, The results were real, palpable, and rewarding in terms of your own good feelings about yourself and your grade point! However, we must think holistically and consider the entire effort, the total enterprise of our teaching which is to help our students see the value of what they are doing, to build their own fabric or structure of support, for all such efforts have to result in a very personal internalization of the mathematics under study for each student.

How Might We Proceed?

We need walls for support, for definition, for separation where appropriate, and for guidance in the hallways which help us with a modest amount of structure in our learning and lead to new disciplines, places, and learning opportunities. However, we do not need walls to limit and keep out new ideas as well as possible mathematics or to keep us from seeing and moving to new approaches in teaching and learning mathematics.

Key elements of student discovery could be to build a wall just in time, develop structure as needed, in an accepting mode in which ideas fit and mesh and whole collections of notions become a theory and a useful set of tools, e.g., matrix transformation, rate of change, accumulation, counting without counting, event space and probability, etc. Posing a problem, as we have done many times in our own teaching career (e.g., [15]), which causes students to seek out building materials for their structural wall of understanding and foundation for mathematical action is one path we could take.

Accepting that it is good to be "off the wall" on occasion, to liven up the learning is also good. Bringing a fresh, different approach is good for both teacher and student. We have been successful, both in our own teaching and in getting others to try this approach, with engaging students as consultants to a client - often the client is the teacher acting as a client - to solve a problem [13]. Students extract the nature of a problem and information from the client, create a set of assumptions, construct mathematical model, and communicate to the client (in one voice) and to the senior staff of the consulting agency (in a different voice). No wall of expectation or restriction bounds the students. They roam freely through their known mathematics and experiences and sometimes produce new approaches we, in our tradition, could not imagine. We have found this to be true with a problem [16] we have used many times in many settings for over 35 years. We are always amazed at the different approaches students take when we do not offer them the wall of restriction afforded by the chapter topic of the text in which we often assign problems or stick to the rigid litany of traditional problems.

We suggest that teachers of mathematics step back and look for walls in their professional life and reflect on which ones are good and supportive and which ones are restrictive and narrowing. Then consider actions to handle the walls around us and take our students on a journey not restricted by the bad aspects of walls, but enhanced by the good attributes of walls.

Conclusion

Robert Frost ends his poem, *Mending Wall* with these words [4, p. 48]:

> Good fences make good neighbors.

While he has tried to define the purpose of a good fence or wall in the poem, as in all poetry it is up to the reader to take the partially fulfilled images of the poet and make something personal with them. So too, we must make personal our understanding of the many images of walls for us in our life as teachers of mathematics. In doing so we reflect upon our calling and help ourselves focus on the real opportunities we have to touch and inform the understanding, views, and lives of our students.

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